

# Hadron decay amplitudes from $B \rightarrow K\pi$ and $B \rightarrow \pi\pi$ decays

<sup>1,2</sup>Xiao-Gang He\* and <sup>3</sup>Bruce H. J. McKellar<sup>†</sup>

<sup>1</sup>*Department of Physics, Peking University, Beijing*

<sup>2</sup>*NCTS/TPE, Department of Physics,  
National Taiwan University, Taipei*

<sup>3</sup> *School of Physics, University of Melbourne, Parkville, Vic 3052*

(Dated: 10 June, 2005)

## Abstract

One can analyze the hadronic decay amplitudes for  $B \rightarrow PP$  decays using flavor SU(3) symmetry in different ways, such as the algebraic and quark diagram approaches. We derive specific relations between these two sets of amplitudes. In the Standard Model the leading hadronic decay amplitudes depend on only five independent parameters which can be determined using recent experimental data on the branching ratios and CP violating asymmetries of  $B \rightarrow K\pi$  and  $B \rightarrow \pi\pi$ . We find, however, that the leading amplitudes provide a best fit solution with a large  $\chi^2$ , which cannot therefore be regarded as a good fit. Keeping sub-leading terms, makes it possible to have a reasonable minimal  $\chi^2$ . We also find that in general the color suppressed decay amplitude is comparable with the color allowed amplitude, contrary to expectations.

PACS numbers: 13.25.Hw, 12.15.Ji, 14.40.Nd

---

\* hexg@phys.ntu.edu.tw.

<sup>†</sup> mckellar@physics.unimelb.edu.au

## I. INTRODUCTION

Recently the Babar and Belle collaborations have measured direct CP violation in  $\bar{B}^0 \rightarrow K^- \pi^+$  with consistent results which average to  $A_{CP}(K^- \pi^+) = -0.109 \pm 0.019$ . [1, 2]. They have also given new, precision determinations of the branching ratios of  $B \rightarrow K\pi$  and  $B \rightarrow \pi\pi$ , which are compiled in [3], and are given in Table I. In the table we also list other CP violating variables although they are not as precisely measured as the branching ratios. These results, and those for other two body  $B$  decays, show that the study of two body charmless  $B$  decays has entered a precision era. They can be used to understand the dynamics of  $B$  decays and CP violation in the Standard Model.

Decay channel	$BR \times 10^6$	$A_{CP}$	$S_f$
$\bar{K}^0 \pi^-$	$24.1 \pm 1.3$	$-0.02 \pm 0.034$	–
$K^- \pi^0$	$12.1 \pm 0.8$	$0.04 \pm 0.04$	–
$K^- \pi^+$	$18.2 \pm 0.8$	$-0.109 \pm 0.019$	–
$\bar{K}^0 \pi^0$	$11.5 \pm 1.0$	$0.09 \pm 0.14$	$0.34 \pm 0.28$
$\pi^- \pi^0$	$5.5 \pm 0.6$	$-0.02 \pm 0.07$	–
$\pi^- \pi^+$	$4.5 \pm 0.4$	$0.37 \pm 0.10$	$-0.50 \pm 0.12$
$\pi^0 \pi^0$	$1.45 \pm 0.28$	$0.28 \pm 0.39$	–

TABLE I: Experimental data on  $B \rightarrow K\pi, \pi\pi$ . The normalizations of  $S_f$  are  $\sin(2\beta)$  and  $\sin(2\alpha)$  in the cases where tree and penguin amplitudes are neglected for  $\bar{K}^0 \pi^0$  and  $\pi^+ \pi^-$ , respectively.

The decay amplitudes for  $B \rightarrow K\pi, \pi\pi$  can be parameterized according to SU(3) (or isospin) symmetry through the equivalent quark diagram or algebraic representations. In these ways, detailed in [4, 5, 6, 7], the decay amplitudes in these and other two body charmless  $B$  decays are related. With enough information, the parameters can be completely fixed. These decay amplitudes, or the equivalent parameters, can also be estimated using various different theoretical approaches. In this work we will carry out our analysis as model independently as possible by using flavor symmetries to study the implications of the measured branching ratios and CP asymmetries in  $B \rightarrow K\pi, \pi\pi$  for the hadronic parameters. Were the parameters to be well determined by the data, one could regard them as features to be explained by attempts to calculate the hadronic matrix elements.

There are many recent studies for  $B \rightarrow K\pi$  and  $B \rightarrow \pi\pi$  decays emphasizing on the determination of the CKM matrix elements and implications for new physics beyond the SM using different approaches[7, 8, 9]. We take a different approach in this analysis by taking CKM parameters as the known ones determined from other data[11], and emphasizes on the determination of the leading and sub-leading contributions to the hadronic parameters in the SM using the most recent data. We first show that different approaches based on SU(3) flavor symmetry are completely equivalent when appropriate terms are taken into account, and obtain specific relations for amplitudes used in diagram and algebraic approaches. We then use available data to determine related hadronic amplitudes. We find that the leading amplitudes provide a best fit solution with a large  $\chi^2$ , more than 2 per degree of freedom, which cannot be regarded as good fit. Keeping the sub-leading terms, makes it possible to fit the data. We also find that in general the color suppressed decay amplitude is comparable with the color allowed amplitude.

Since the first draft of this paper, several articles have been written on related subjects[12, 13, 14], and similar results have been obtained although different authors emphasizes different features of the analysis.

## II. PARAMETRIZATION OF THE DECAY AMPLITUDES

There are several different ways of parameterizing SU(3) decay amplitudes for  $B \rightarrow PP$  decays. We start our analysis by showing that they are all equivalent when the appropriate terms are included.

In the SM, the decay amplitudes for  $B \rightarrow K\pi, \pi\pi$  can be parameterized by separating the terms according to the relevant products of CKM matrix elements:

$$\begin{aligned} A_{B \rightarrow K\pi} &= V_{ub}V_{us}^* T_{K\pi} - V_{tb}V_{ts}^* P_{K\pi}, \\ A_{B \rightarrow \pi\pi} &= V_{ub}V_{ud}^* T'_{\pi\pi} - V_{tb}V_{td}^* P'_{\pi\pi}, \end{aligned} \tag{1}$$

where  $V_{ij}$  are the CKM matrix elements which in general contain CP violating phases. The amplitudes  $T_{PP}(T'_{PP})$  and  $P_{PP}(P'_{PP})$  are hadronic matrix elements which in general contain CP conserving final state interaction phases. The primed  $(T', P')_{PP}$  and the un-primed  $(T, P)_{PP}$  amplitudes are equal in the flavor SU(3) symmetry limit. In the above we have used the unitarity of the CKM matrix to eliminate terms proportional to  $V_{cb}V_{cs}^*$  and  $V_{cb}V_{cd}^*$

in favor of the above two terms.

In the SM the quark level Hamiltonian[15]  $H$ , expanded in dimension six operators,

$$H = \frac{G_F}{\sqrt{2}} \left[ V_{ub}V_{uq}^* (c_1 O_1 + c_2 O_2) - \sum_j V_{jb}V_{jq}^* \sum_{i=3}^{10} c_i^j O_i \right],$$

requires the hadronic matrix elements  $T_{PP}$  and  $P_{PP}$  transform under  $SU(3)$  as  $\bar{3}$ , 6 and  $\bar{15}$ . In the above  $c_i$  are the Wilson coefficients of the operators  $O_i$  which have been calculated to next-to-leading order in QCD corrections. We will use the values calculated in the NDR regularization scheme at  $\mu = m_b$  given in Ref.[15].

The amplitude  $T_{PP}$  is dominated by the operators  $O_{1,2}$ , which generate, in quark diagram language, the “color allowed”  $T$  and “color suppressed”  $C$  amplitudes, containing  $\bar{3}$ , 6 and  $\bar{15}$  irreducible amplitudes. The  $P_{PP}$ , called the “penguin” amplitude, is generated at the loop level and contains strong and electroweak penguin contributions. The strong penguin induces only a  $\bar{3}$  amplitude, but the electroweak penguin, dominated by the operators  $O_{9,10}$ , induces all  $\bar{3}$ , 6 and  $\bar{15}$  amplitudes. As far as the  $SU(3)$  group structure is concerned, the electroweak penguin operators are proportional to  $3O_{1,2}/2 - (1/2)O_{3,4}$ . One can easily obtain the electroweak penguin irreducible amplitude by grouping the part proportional to  $O_{3,4,5,6}$  into a strong penguin like operator and the rest into a tree like operator.

The decay amplitudes can be parameterized according to the  $SU(3)$  irreducible amplitudes[7], separating the “annihilation” amplitudes,  $A_{\bar{3},\bar{15}}^{T,P}$ , in which the initial quarks are annihilated in the weak interaction from the amplitudes  $C_{\bar{3},6,\bar{15}}^{T,P}$ , in which one of the initial quarks is preserved. There is also an  $A_6$  amplitude, which has the same coefficients as the  $C_6$  amplitude and can therefore be absorbed into it. We list the “tree” amplitudes for  $B \rightarrow K\pi, \pi\pi$  in Table II.

In the quark diagram approach, the decay amplitudes for various decay modes are parameterized by the  $T$ ,  $C$ ,  $P$ ,  $A$ ,  $E$  and  $P_A$  amplitudes which parameterize the color allowed, color suppressed, the flavor triplet strong penguin, the annihilation, exchange and penguin annihilation amplitudes. The details for each decay amplitudes in terms of the above diagram amplitudes can be found in Ref.[4, 5]. In the diagram approach neglecting annihilation contributions, the amplitude for  $B^- \rightarrow \pi^- \bar{K}^0$  vanishes which implies in the algebraic approach  $\delta^T = C_{\bar{3}}^T - C_6 - C_{\bar{15}}^T = 0$ . Comparing the decay modes  $B^- \rightarrow \pi^- \pi^0$ , and  $\bar{B}^0 \rightarrow \pi^+ K^-$ ,  $\pi^0 \bar{K}^0$ , one can identify,  $8V_{ub}V_{us}^* C_{\bar{15}} = (T + C)e^{-i\gamma}$ , and  $4V_{ud}V_{us}^* C_6 = (T - C)e^{-i\gamma}$ . After restoring the annihilation (exchange) contributions, we find the following relations between the

Decay Mode	$SU(3)$ Invariant Amplitude
$T_{\pi^-\pi^0}$	$\frac{8}{\sqrt{2}}C_{15}^T$
$T_{\pi^+\pi^-}$	$2A_{\frac{3}{2}}^T + A_{\frac{15}{2}}^T + C_{\frac{3}{2}}^T + C_6^T + 3C_{15}^T$
$T_{\pi^0\pi^0}$	$\frac{1}{\sqrt{2}}(2A_{\frac{3}{2}}^T + A_{\frac{15}{2}}^T + C_{\frac{3}{2}}^T + C_6^T - 5C_{15}^T)$
$T_{\pi^-\bar{K}^0}$	$3A_{\frac{15}{2}}^T + C_{\frac{3}{2}}^T - C_6^T - C_{15}^T$
$T_{\pi^0K^-}$	$\frac{1}{\sqrt{2}}(3A_{\frac{15}{2}}^T + C_{\frac{3}{2}}^T - C_6^T + 7C_{15}^T)$
$T_{\pi^+K^-}$	$-A_{\frac{15}{2}}^T + C_{\frac{3}{2}}^T + C_6^T + 3C_{15}^T$
$T_{\pi^0\bar{K}^0}$	$-\frac{1}{\sqrt{2}}(-A_{\frac{15}{2}}^T + C_{\frac{3}{2}}^T + C_6^T - 5C_{15}^T)$

TABLE II: The  $SU(3)$  invariant amplitude for  $B \rightarrow \pi\pi, \pi K$  decays. Similar amplitude for the strong and electroweak penguin amplitudes.

algebraic and diagram amplitudes:

$$\begin{aligned}
Te^{-i\gamma} &= V_{ub}V_{us}^*(-A_{\frac{15}{2}} + 2C_6 + 4C_{15}), \quad Ce^{-i\gamma} = V_{ub}V_{us}^*(A_{\frac{15}{2}} - 2C_6 + 4C_{15}), \\
Ae^{-i\gamma} &= 3V_{ub}V_{us}^*A_{\frac{15}{2}}, \quad Ee^{-i\gamma} = 2V_{ub}V_{us}^*(A_{\frac{3}{2}} + A_{\frac{15}{2}}).
\end{aligned} \tag{2}$$

With the above relations one finds that the tree contributions, terms proportional to  $V_{ub}V_{us}^*$ , are equivalent in form for the algebraic and diagram approaches.

When  $\delta^T$  is not equal to zero, there seems to be a conflict in that there are five and four independent variables for the algebraic and diagram amplitudes, respectively. This puzzle is resolved by realizing that the diagram amplitudes listed above has missed a piece of contribution, the penguin contribution with u-quark in the loop (and also a c-quark since we have used the CKM unitarity to eliminate the term proportional to  $V_{cb}V_{cs}^*$  due to c-quark in the loop). Indicating this contribution by  $P_{cu}$ , the charming penguin, and comparing with the algebraic amplitudes, one can identify  $P_{cu}e^{-i\gamma} = V_{ub}V_{us}^*\delta^T$ .

For the strong penguin amplitude, one identifies  $P = V_{tb}V_{ts}^*C_{\frac{3}{2}}^P$ , and  $P_A = V_{tb}V_{ts}^*A_{\frac{3}{2}}^P$ . When electroweak penguin amplitudes are included, one can define a set of parameters  $T^{EW}$ ,  $C^{EW}$ ,  $P^{EW}$ ,  $A^{EW}$ ,  $E^{EW}$  and  $P_A^{EW}$  similar to the previously defined tree quark diagram amplitudes. Here there is no need to introduce an additional electroweak penguin amplitude analogous to the  $P_{cu}$  amplitude because  $P^{EW}$  already includes such a contribution.

With the above relations between the parameters used in the algebraic and diagram approaches, we therefore have shown that the two ways of parameterizing  $B \rightarrow \pi\pi, \pi K$ , the algebraic and diagram approaches, are fully equivalent.

In the SM the tree amplitudes and the electroweak penguin amplitudes are dominated by  $O_{1,2}$  and  $O_{9,10}$  (the operators  $O_{7,8}$  have much smaller Wilson coefficients and can be neglected to a good precision) where the SU(3) invariant amplitudes  $C_6$  and  $C_{\overline{15}}$  originate. Since these operators have the same Lorentz structure and  $O_{9,10} = \frac{3}{2}O_{1,2} - \frac{1}{3}O_{3,4}$ , one finds[7, 16] that  $C_6^P = -(3/2)\kappa^- C_6^T$  and  $C_{\overline{15}}^P(A_{\overline{15}}^P) = (3/2)\kappa^+ C_{\overline{15}}^T(A_{\overline{15}}^T)$ . Here  $\kappa^\pm = (c_9 \pm c_{10})/(c_1 \pm c_2)$ . These relations enable one to reduce the number of independent decay amplitudes, but they, in particular the relation between  $C_6^T$  and  $C_6^P$ , have not been fully exploited in many of the analyses in the literature. There is no simple relation between  $C_3^T(A_3^T)$  and  $C_3^P(A_3^P)$ . Using the above mentioned relations, one finds that

$$T^{EW} + C^{EW} = \frac{3}{2}R\kappa^+(T + C), \quad T^{EW} - C^{EW} = \frac{3}{2}R\kappa^-(T - C), \quad A^{EW} = \frac{3}{2}R\kappa^+A, \quad (3)$$

where  $R = |V_{tb}V_{ts}^*/V_{ub}V_{us}^*|$ .

Because that the amplitude  $A_3^T$  is not simply related to  $A_3^P$  by similar relations for  $C_{6,\overline{15}}$ ,  $E$  and  $E^{EW}$  cannot be simply related. However, for the special case with  $A_3 = 0$ ,  $E^{EW} = \frac{3}{2}\kappa^+E$ ,  $A$  is also related to  $E$  by  $E = 3A/2$ .

The three electroweak amplitudes  $T^{EW}$ ,  $C^{EW}$  and  $P^{EW}$  are usually written in terms of two amplitudes  $P_{EW}$  and  $P_{EW}^C$ , as they are not independent since all originated from the same electroweak penguin operators. To the leading order this relationship is  $T^{EW} = P_{EW}^C$ ,  $C^{EW} = P_{EW}$ , and  $P^{EW} = -P_{EW}^C/3$ .

In the SU(3) limit, the amplitudes for  $B \rightarrow \pi\pi$  can be obtained from the previous amplitudes by an appropriate re-scaling, for the tree amplitudes by  $(V_{ud}^*/V_{us}^*) \approx 1/\lambda$ , and for the strong and electroweak penguin amplitudes by  $r = (V_{td}^*/V_{ts}^*)$ . We summarize the complete set of amplitudes in Table III.

As the amplitudes  $P_{cu}$ ,  $A$ ,  $E$ ,  $E^{EW}$  and  $P_A$  are expected to be smaller than the other amplitudes, one hopes to obtain reasonable description of the relevant data even with their contributions ignored. With this approximation, the analysis is tremendously simplified with only five independent hadronic parameters in the three complex amplitudes,  $T$ ,  $C$ ,  $P$  (one phase of these complex parameters can be absorbed into redefinition of the meson fields). In the following we will first carry out an analysis using this approximation. We find however that these leading amplitudes cannot provide a good description of the data since the resulting fit has too large a minimal  $\chi^2$ .

$A(PP)$	$T$	$C$	$P$	$P_{cu}$	$\kappa^+(T+C)$	$\kappa^-(T-C)$	$A$	$E$	$E^{EW}$	$P_A$
$A(\bar{K}^0\pi^-)$	0	0	-1	$-e^{-i\gamma}$	0	0	$-e^{-i\gamma} - \frac{3R}{2}\kappa^+$	0	0	0
$\sqrt{2}A(K^-\pi^0)$	$-e^{-i\gamma}$	$-e^{-i\gamma}$	-1	$-e^{-i\gamma}$	$-\frac{3R}{2}$	0	$-e^{-i\gamma} - \frac{3R}{2}\kappa^+$	0	0	0
$A(K^-\pi^+)$	$-e^{-i\gamma}$	0	-1	$-e^{-i\gamma}$	$-\frac{3R}{4}$	$\frac{3R}{4}$	0	0	0	0
$\sqrt{2}A(\bar{K}^0\pi^0)$	0	$-e^{-i\gamma}$	1	$e^{-i\gamma}$	$-\frac{3R}{4}$	$-\frac{3R}{4}$	0	0	0	0
$A(PP)$	$T'$	$C'$	$P'$	$P'_{cu}$	$\kappa^+(T'+C')$	$\kappa^-(T'-C')$	$A'$	$E'$	$E'^{EW}$	$P'_A$
$\sqrt{2}A(\pi^-\pi^0)$	$-\frac{1}{\lambda}e^{-i\gamma}$	$-\frac{1}{\lambda}e^{-i\gamma}$	0	0	$-\frac{3R}{2}r$	0	0	0	0	0
$A(\pi^-\pi^+)$	$-\frac{1}{\lambda}e^{-i\gamma}$	0	-r	$-\frac{1}{\lambda}e^{-i\gamma}$	$-\frac{3R}{4}r$	$\frac{3R}{4}r$	0	$-\frac{1}{\lambda}e^{-i\gamma}$	$-rR$	-r
$\sqrt{2}A(\pi^0\pi^0)$	0	$-\frac{1}{\lambda}e^{-i\gamma}$	r	$\frac{1}{\lambda}e^{-i\gamma}$	$-\frac{3R}{4}r$	$-\frac{3R}{4}r$	0	$\frac{1}{\lambda}e^{-i\gamma}$	$rR$	r

TABLE III: The quark diagram amplitudes for  $B \rightarrow K\pi, \pi\pi$ .  $R = |V_{tb}V_{ts}^*/V_{ub}V_{us}^*|$ ,  $\lambda = V_{us}^*/V_{ud}^*$  and  $r = V_{td}^*/V_{ts}^*$ . In the  $SU(3)$  limit  $(T, C, P, A, E, E^{EW}, P_A) = (T', C', P', A', E', E'^{EW}, P'_A)$ . We have written the notation in the above  $P(P')$  and  $PA(P'_A)$  for the combined strong and electroweak penguin amplitudes  $P - P_{EW}^C/3$  and  $P_A + P_A^{EW}$ .

### III. THE $B \rightarrow K\pi$ DATA AND THE HADRONIC PARAMETERS

At present there are 5 well established measurements for  $B \rightarrow K\pi$  decays: the four branching ratios and the CP asymmetry in  $\bar{B}^0 \rightarrow K^-\pi^+$ . Using these data and taking the CKM matrix elements determined from various experimental data, one can determine the hadronic parameters. We take the central values for the CKM parameters  $s_{12} = 0.2243$ ,  $s_{23} = 0.0413$ ,  $s_{33} = 0.0037$  and the CP violating phase  $\gamma(\delta_{13}) = 60^\circ$  given by Ref.[11]. For the central experimental data we obtain two solutions for the amplitudes  $C$  and  $T$

$$\begin{aligned}
1) \quad & T = 1.018 e^{3.092i}; \quad C = 1.158 e^{0.0916i}. \\
2) \quad & T = 1.016 e^{-2.978i}; \quad C = 1.154 e^{0.0070i}.
\end{aligned} \tag{4}$$

It is remarkable that there are any solutions. The magnitude of the amplitudes is almost the same, for the two solutions, while the phases have changed.

In the analysis we have normalized the amplitudes to the amplitude of  $A(\bar{K}^0\pi^-)$  and to obtain the physical numbers they should be multiplied by a factor  $\sqrt{B^{exp}(\bar{K}^0\pi)16\pi m_B \Gamma_{total}^B}$  with  $B^{exp} = 24.1 \times 10^{-6}$ . Since  $P$  is determined by  $A(\bar{K}^0\pi^-)$ , we set  $P = 1$  in this case. In the calculations we have also taken into account the  $\bar{B}^0$  and  $B^-$  lifetime, and (later) the  $B \rightarrow K\pi$  and  $B \rightarrow \pi\pi$  phase space differences.

Note that, in each of the solutions,  $C$  and  $T$  are almost real. This fits with the intuition that the final state mesons have large energies and the final state interactions which generate the phase in  $C$  and  $T$  would be expected to be weak, leading to small phases. However the ratios  $|C/T|$ ,  $|T/P|$  and  $|C/P|$  are of order one, which was also found in refs [8, 9], and which is in contradiction with expectations from various theoretical calculations. This poses a problem for our ability to provide a theoretical basis for the observed amplitudes.

Using the hadronic parameters determined above, one can predict the CP asymmetries in other  $B \rightarrow K\pi$  decays. Since  $B^- \rightarrow \bar{K}^0\pi^-$  has only a  $P$  amplitude, no CP asymmetry can be generated in this decay. There are non-zero asymmetries in the other two decays. We find that for solution 1),  $A_{CP}(K^-\pi^0) = 0.267$ ,  $A_{CP}(\bar{K}^0\pi^0) = -0.006$ ,  $S_{\bar{K}^0\pi^0} = -0.375$ . For solution 2),  $A_{CP}(K^-\pi^0) = -0.266$ ,  $A_{CP}(\bar{K}^0\pi^0) = -0.198$ ,  $S_{\bar{K}^0\pi^0} = -0.378$ . These values are different to the central values of the data and the two solutions can be distinguished in the near future. We note that the predicted CP asymmetry for  $\bar{B}^0 \rightarrow K^-\pi^0$  for both solutions are much larger in size than the central value of the data. This also poses another potential problem for the solutions.

One can also include the direct CP asymmetry data into a  $\chi^2$  fit. If we use all  $B \rightarrow K\pi$  data, the best fit values for the parameters and the resulting branching ratios (in unit  $10^{-6}$ ) and CP asymmetries are given by

$$\begin{aligned}
P &= 0.999, \quad T = 0.127 e^{-0.533i}, \quad C = 0.260 e^{0.295i}. \\
B(\bar{K}^0\pi^-) &= 24.06, \quad B(K^-\pi^0) = 12.30, \\
B(K^-\pi^+) &= 10.41, \quad B(\bar{K}^0\pi^0) = 18.47; \\
A_{CP}(K^-\pi^+) &= -0.104, \quad A_{CP}(K^-\pi^0) = 0.019, \\
A_{CP}(\bar{K}^0\pi^0) &= -0.150, \quad S_{\bar{K}^0\pi^0} = 0.339.
\end{aligned} \tag{5}$$

Note that here  $P$  is also a fitting parameter. The  $\chi^2_{min}$  is 4.68 for 4 degrees of freedom which represents a reasonable fit.

If all the CP asymmetries, direct and time-dependent, are measured to a good precision in the future, one can also check the consistency of CKM parameters determined from other data by taking  $\rho$  and  $\eta$  as unknown and letting them be determined from  $B \rightarrow K\pi$  data[7, 17].

In fact the analysis carried out above using the well measured branching ratios and CP asymmetry in  $K^-\pi^+$  mode already constrains the allowed range for  $\gamma$  since for certain



values of  $\gamma$ , there are no solutions for the hadronic parameters. For example  $\gamma$  in the interval between  $54.5^\circ$  to  $100^\circ$  is allowed, but the intervals between  $38.5^\circ$  to  $54.5^\circ$  and  $105^\circ$  to  $145.5^\circ$  are not allowed.

The above analysis shows that the leading SU(3) amplitudes can provide a good description of  $B \rightarrow K\pi$  data if one allows a larger than expected  $|C/T|$  ratio.

#### IV. PREDICTIONS AND PROBLEMS FOR $B \rightarrow \pi\pi$ DECAYS

In the SU(3) limit, once the parameters  $T$ ,  $C$  and  $P$  have been determined from  $B \rightarrow K\pi$  decays, predictions can be made for  $B \rightarrow \pi\pi$  decays. An interesting prediction is the CP asymmetry in  $\bar{B}^0 \rightarrow \pi^-\pi^+$  which can be made without knowing the specific values of the amplitudes  $T$ ,  $C$  and  $P$ . From Table III and the fact that in the SM  $Im(V_{ub}V_{us}^*V_{tb}V_{ts}) = -Im(V_{ub}V_{ud}^*V_{tb}V_{td})$ , one has:  $\Delta_{K^-\pi^+}^{\bar{B}^0} = -\Delta_{\pi^-\pi^+}^{\bar{B}^0}$ . Here  $\Delta_{PP}^B = \Gamma_{PP}^B - \Gamma_{\bar{P}\bar{P}}^B$ . This relation leads to[18]

$$A_{CP}(\pi^-\pi^+) = (-1)A_{CP}(K^-\pi^+)\frac{B(K^-\pi^+)}{B(\pi^-\pi^+)}. \quad (6)$$

There are  $SU(3)$  breaking effects which modify the above relation. Using the QCD factorization method to make an estimate about the SU(3) breaking effects due to meson decay constants and light cone distribution for different mesons, the factor  $-1$  in the above equation is changed to[18]  $-0.9$ . One would predict a CP asymmetry in  $\bar{B}^0 \rightarrow \pi^-\pi^+$  of  $0.39 \pm 0.08$  which is consistent with the experimental value[3].

We look forward to a high precision measurement of  $A_{CP}(\pi^-\pi^+)$ , which will provide a very direct test of the SM.

Given the values of  $T$ ,  $C$  and  $P$  more predictions can be made for  $B \rightarrow \pi\pi$  decays. However, the values obtained in eqs. (4) and (5) would imply branching ratios of  $B^- \rightarrow \pi^-\pi^0$ ,  $\bar{B}^0 \rightarrow \pi^-\pi^+$ ,  $\pi^0\pi^0$  which are too large compared with experimental data. This is mainly due to the  $T$  and  $C$  parameters, as determined from the  $B \rightarrow K\pi$  data, being too large for the  $\pi\pi$  decays. The simple parametrization and the present experimental data are not consistent using the same set of leading amplitudes to explain both the  $B \rightarrow K\pi$  and  $B \rightarrow \pi\pi$  data.

The use of central values to determine the hadronic parameters is, of course, too restrictive. It is possible that in the ranges allowed by the experimental errors, a consistent solution

can be found. We therefore carried out a  $\chi^2$  fit to determine the parameters. We consider the case with SU(3) symmetry taking the four  $B \rightarrow K\pi$ , the three  $\pi\pi$  branching ratios, and the direct CP asymmetry  $A_{CP}(\bar{B}^0 \rightarrow K^-\pi^+)$  as the input data points to determine the best fit values for the five hadronic parameters. We obtain

$$P = 0.971, \quad T = 0.090 e^{-2.44i}, \quad C = 0.076 e^{-1.54i}. \quad (7)$$

We find that with this set of parameters the resulting branching ratios and CP asymmetry are within the two standard deviation ranges of the data. However, the  $\chi^2$  at the minimum is 8.42 for 3 degrees of freedom which is rather high. This indicates that there is a potential problem for the leading parametrization to explain all  $B \rightarrow K\pi$ ,  $\pi\pi$  data

Including all the branching ratio and CP asymmetry data in Table I in the fitting, we obtain

$$P = 0.941, \quad T = 0.086 e^{-2.585i}, \quad C = 0.0817 e^{2.65i}. \quad (8)$$

With this set of hadronic parameters, the branching ratios and asymmetries are

$$\begin{aligned} B(\bar{K}^0\pi^-) &= 21.32, \quad B(\pi^-\pi^0) = 4, 34, \\ B(K^-\pi^0) &= 11.19, \quad A_{CP}(K^-\pi^0) = -0.015, \\ B(\bar{K}^0\pi^0) &= 9.753, \quad A_{CP}(\bar{K}^0\pi^0) = -0.053 \\ B(K^-\pi^+) &= 20.13, \quad A_{CP}(K^-\pi^+) = -0.097, \\ B(\pi^0\pi^0) &= 1.61, \quad A_{CP}(\pi^0\pi^0) = 0.410, \\ B(\pi^-\pi^+) &= 4.88, \quad A_{CP}(\pi^-\pi^+) = 0.388, \\ S_{\bar{K}^0\pi^0} &= 0.691, \quad S_{\pi^+\pi^-} = -0.78. \end{aligned} \quad (9)$$

The  $\chi^2_{min}$  increased to about 26.5 for 9 degrees of freedom. This is similar to the fitting quality for the case of eq. (ssss).

We conclude that the leading order parametrization has a problem with the present data. That could be due to the smaller sub-leading terms of the SM which we neglected in constructing the leading amplitudes parametrization playing a more important role than expected, or to SU(3) breaking effects. It may be due to the quality of the data. And it may also be due to physics beyond the SM[8, 9, 10]. Before making any claims for the existence of new physics beyond the SM, one must make sure that the SM contributions, leading and

also sub-leading included, cannot account for the data. In the next section we analyze the effects of the sub-leading terms which have been neglected in the previous analysis.

## V. EXPANDING THE PARAMETER SET

A number of approximations have been made to obtain the leading amplitude parametrization with just five independent hadronic variables: 1)  $\delta^T = 0$ , 2) neglect of the annihilation amplitude  $Ae^{-i\gamma} = 3V_{ub}V_{us}^*A_{15}^T$ , the exchange amplitude  $Ee^{-i\gamma} = 2V_{ub}V_{us}^*(A_3^T + A_{15}^T)$ , and penguin annihilation amplitude  $P_A = A_3^P$ , and 3) SU(3) symmetry hold for the hadronic matrix elements in  $B \rightarrow PP$  decays.

The first and the second approximations can be tested experimentally to some degree. If  $\delta^T = 0$  and  $A = 0$ , CP asymmetry in  $B^- \rightarrow \bar{K}^0\pi^-$  will be very small. Since the tree amplitude for  $B^- \rightarrow K^-K^0$  has the same form as that for  $B^- \rightarrow \bar{K}^0\pi^-$ , the assumption of  $\delta^T = 0$  and  $A = 0$  implies a very small branching ratio for  $B^- \rightarrow K^-K^0$  (penguin contribution to this decay mode is suppressed). Experimentally, a non-zero CP asymmetry for  $B^- \rightarrow \bar{K}^0\pi^-$  and a non-zero branching ratio for  $B^- \rightarrow K^-K^0$  decay has not been established,  $\delta^T \neq 0$  and  $A \neq 0$  are not required. The smallness of  $E$  can be tested by  $\bar{B}^0 \rightarrow K^+K^-$  since its tree amplitude is proportional to  $E$ . At present a non-zero amplitude for this mode has not been established either. We however find that the experimental upper bounds obtained for these decays can constrain the parameters  $A$ ,  $E$  and  $P_{cu}$ . The tree amplitude for  $\bar{B}^0 \rightarrow \bar{K}^0K^0$  also depends on  $P_{cu}$  and other small parameters, the upper bound on this mode therefore can also provide constraint on the parameters. It is not possible to have a good test of the smallness of  $P_A$  directly in  $B \rightarrow KK$  modes since it is suppressed by a factor of  $\lambda r$ . We now analyze whether the restoration of these small amplitudes can improve the fit.

The contributions from  $A$ ,  $E$ ,  $E^{EW}$  and  $P_A$  are annihilation in nature, their sizes are expected to be smaller than the amplitudes  $T$ ,  $C$  and  $P$ . If the final results from fit ended up with comparable size for these parameters, one should not regard the fit a good one. The parameter  $P_{cu}$  is a penguin amplitude in nature, it should be compared with the amplitude  $\sim \lambda^2 P$ , where the pre-factor  $\lambda^2$  takes care of the CKM suppression of  $P_{cu}$  compared with  $P$ .

To study the effects of  $A$  and  $E$ , we use two independent parameters  $\epsilon$  and  $\tau$  defined as  $\tau e^{-i\gamma} = V_{ub}V_{us}^*A_{15}^T$  and  $\epsilon e^{-i\gamma} = V_{ub}V_{us}^*A_3^T$  which have definitive SU(3) irreducible structure.

We find that the minimal  $\chi^2$  can be improved. To have specific idea about how these new sub-leading parameters affects the decays, we studied three cases with only one of the parameters  $P_{cu}$ ,  $\tau$  and  $\epsilon$  to be non-zero separately, and fitting the measured branching ratios for  $B \rightarrow K\pi, \pi\pi$  and direct CP asymmetries for  $B \rightarrow K^+\pi^-, \bar{K}^0\pi^0, K^-\pi^0, \pi^+\pi^-, \pi^0\pi^0$ . We also include information about the branching ratios for [3]  $B \rightarrow K^-K^0 (< 2.4 \times 10^{-6})$ ,  $\bar{K}^0\bar{K}^0 ((1.19 \pm 0.4) \times 10^{-6})$ ,  $K^-K^+ (< 0.6 \times 10^{-6})$  into the fit. For these modes, the leading amplitudes are dominated by tree amplitudes with  $A(\bar{K}^0K^-) \approx -(3\tau + P_{cu})e^{-i\gamma}/\lambda$ ,  $A(\bar{K}^0K^0) \approx -(2\epsilon - 3\tau + P_{cu})e^{-i\gamma}/\lambda$ , and  $A(K^-K^+) \approx -2(\epsilon + \tau)e^{-i\gamma}/\lambda$ . For the two modes having just upper bounds, we treat their central values to be zero and taking the 68% c.l. range as errors in the fit. We then predict the values for  $S_{\pi^0\bar{K}^0}$ ,  $S_{\pi^+\pi^-}$  and  $A_{CP}(\bar{K}^0\pi^0)$ .

Without the sub-leading parameters, the  $\chi^2_{min}$  without the  $B \rightarrow KK$  data would be about 26. Such a fit cannot be considered to be a good fit. With the sub-leading parameters,  $B \rightarrow KK$  can happen. Including information on  $B \rightarrow KK$  branching ratios into the fit, we find that the  $\chi^2_{min}$  are 12, 8, 17 for: i) Only  $P_{cu} \neq 0$ ; ii) only  $\tau \neq 0$ ; and iii)  $\epsilon \neq 0$ . The  $\chi^2_{min}$  is significantly reduced. In each of the fitting above the degrees of freedom is 8. The cases i) and ii) can be regarded as reasonable fits. We list the best fit values for the relevant quantities for the cases i) and ii) in the following.

For i), we have

$$\begin{aligned}
P &= 0.953, \quad T = 0.135e^{-2.806i}, \quad C = 0.061e^{1.924i}, \quad P_{cu} = 0.050e^{0.057i}, \quad \tau = 0 \quad \epsilon = 0, \\
B(\bar{K}^0\pi^-) &= 23.11, \quad B(\pi^-\pi^0) = 5.32, \\
B(K^-\pi^0) &= 12.04, \quad A_{CP}(K^-\pi^0) = 0.027, \\
B(\bar{K}^0\pi^0) &= 9.31, \quad A_{CP}(\bar{K}^0\pi^0) = -0.083 \\
B(K^-\pi^+) &= 19.32, \quad A_{CP}(K^-\pi^+) = -0.098, \\
B(\pi^0\pi^0) &= 1.34, \quad A_{CP}(\pi^0\pi^0) = 0.751, \\
B(\pi^-\pi^+) &= 4.59, \quad A_{CP}(\pi^-\pi^+) = 0.397, \\
B(\bar{K}^0K^0) &= 1.26, \quad B(K^-K^0) = 1.26, \quad B(K^-K^+) \approx 0, \\
S_{\bar{K}^0\pi^0} &= 0.713, \quad S_{\pi^+\pi^-} = -0.874, \quad A_{CP}(\bar{K}^0\pi^-) = 0.005,
\end{aligned} \tag{10}$$

and for case ii), we have

$$\begin{aligned}
P &= 0.928, \quad T = 0.111e^{-0.487i}, \quad C = 0.071e^{0.659i}, \quad P_{cu} = 0, \quad \tau = 0.017e^{2.81i} \quad \epsilon = 0, \\
B(\bar{K}^0\pi^-) &= 23.80, \quad B(\pi^-\pi^0) = 5.70, \\
B(K^-\pi^0) &= 11.55, \quad A_{CP}(K^-\pi^0) = 0.017, \\
B(\bar{K}^0\pi^0) &= 9.86, \quad A_{CP}(\bar{K}^0\pi^0) = -0.087 \\
B(K^-\pi^+) &= 19.17, \quad A_{CP}(K^-\pi^+) = -0.110, \\
B(\pi^0\pi^0) &= 1.36, \quad A_{CP}(\pi^0\pi^0) = 0.669, \\
B(\pi^-\pi^+) &= 4.36, \quad A_{CP}(\pi^-\pi^+) = 0.370, \\
B(\bar{K}^0K^0) &= 1.27, \quad B(K^-K^0) = 0.06, \quad B(K^-K^+) = 0.03, \\
S_{\bar{K}^0\pi^0} &= 0.626, \quad S_{\pi^+\pi^-} = 0.504, \quad A_{CP}(\bar{K}^0\pi^-) = 0.027.
\end{aligned} \tag{11}$$

In the above two cases a small CP asymmetry for the mode  $B^- \rightarrow \bar{K}^0\pi^-$  is developed because the best fit values for  $P_{cu}$  and  $\tau$  are complex. The predicted time dependent CP asymmetry  $S_{\pi^+\pi^-}$  are opposite in sign with case i) having the same sign as the present experimental data. The two types of solutions are be easily distinguished by a definitive measurement of  $S_{\pi^+\pi^-}$ .

The numerical values obtained for the cases i) and ii) are within expectations that  $\tau$  is smaller than  $T$ , and  $P_{cu}$  is of order  $\lambda^2 P$ . We note that the above cases the ratio for  $|C/T|$  is still large which may be completely due to low energy hadronic physics with in the SM. We conclude that when sub-leading contributions are included, SM can provide a good fit to  $B \rightarrow K\pi$  and  $B \rightarrow \pi\pi$  data. No new physics beyond the SM is called for at present. It is obvious that if the small parameters are simultaneously kept non-zero, better fit can be obtained.

## VI. SU(3) BREAKING EFFECTS

We have carried out all the above analysis with the assumption of SU(3) symmetry. With SU(3) breaking effects taken into account the set of parameters for  $B \rightarrow K\pi$  and  $B \rightarrow \pi\pi$  can be different. In the algebraic and diagram approaches described earlier, the SU(3) breaking effects can be systematically included by inserting at appropriate places the quark mass terms which will introduces many new unknown parameters. We will not attempt to

carry out a general analysis here, but to simplify the problem by assuming that the  $B \rightarrow K\pi$  amplitudes are scaled by a factor  $f_K/f_\pi$  compared with  $B \rightarrow \pi\pi$  amplitudes.

We argue that the above re-scaling factor should have taken into account the leading effects of SU(3) breaking. An intuitive picture can be obtained from pQCD calculations of the decay amplitudes. In pQCD calculations decay amplitudes proportional to  $f_B f_\pi f_\pi$  and  $f_B f_\pi f_K$  for  $B \rightarrow \pi\pi$  and  $B \rightarrow K\pi$  decays, and therefore the re-scaling factor is  $f_K/f_\pi$ . There are other places where SU(3) breaking effects may come from, for example, the difference between the light-cone distribution amplitudes for the pion and kaon. Neglecting the later type of SU(3) breaking effects, the analysis can be carried out in the same way as previously done since no new adjustable parameter is introduced.

Using the same data points as what used for the analysis obtaining eq.(7), we find that the fit improved slightly with  $\chi^2_{min}$  reduced to 7.3 from 8.4. The best fit values for the hadronic parameters are

$$P = 0.978, \quad T = 0.119e^{-2.60i}, \quad C = 0.100e^{-1.57i}, \quad (12)$$

Using all available data from  $B \rightarrow K\pi$ ,  $\pi\pi$ ,  $\chi^2_{min}$  is reduced to 22.4 from 26.5 with

$$P = 0.944, \quad T = 0.116e^{-2.70i}, \quad C = 0.103e^{2.496i}. \quad (13)$$

The above analysis shows that the simple re-scaling on the decay amplitudes can improve the fit, but cannot completely solve the problem, and still allows for sub-leading amplitudes to play an important role.

An extreme SU(3) breaking scenario is the case where  $B \rightarrow K\pi$  and  $B \rightarrow \pi\pi$  decays are treated independently. We have seen that if one just fits  $B \rightarrow K\pi$  data, the leading amplitudes can account for data, but predict too large branching ratios for  $B \rightarrow \pi\pi$  decays. We now study the consequences of just fitting the  $B \rightarrow \pi\pi$  data using the leading amplitudes. We find that the five parameters  $T'$ ,  $C'$  and  $P'$  have no problem in fitting the data on branching ratios. To determine the parameters, at least two more points of experimental data are needed which we choose to take the direct and time dependent CP asymmetries  $A_{CP}$  and  $S_{\pi\pi}$  for  $B \rightarrow \pi^-\pi^+$ . With the central values of the data, we find two solutions

$$\begin{aligned} 1) & P = 0.576, \quad T = 0.098 e^{-2.17i}, \quad C = 0.077 e^{-1.12i}, ; \\ 2) & P = 0.506, \quad T = 0.098 e^{-2.21i}, \quad C = 0.083 e^{2.73i}. \end{aligned} \quad (14)$$

These solutions predict  $A_{CP}(\pi^0\pi^0)$  to be  $-0.60$  and  $0.18$ , respectively, and the second solution is closer to the experimental central value.

These parameter values are quite different to those in eq. (4) and in eq. (5), and if we apply them to  $B \rightarrow K\pi$ , the branching ratios obtained for  $B \rightarrow K\pi$  are too small.

We also carried out another alternative fit by using all  $B \rightarrow \pi\pi$ . This fit obtains  $\chi^2_{min} = 0.34$  with

$$P = 0.487, \quad T = 0.099e^{-2.23i}, \quad C = 0.084e^{2.88i}. \quad (15)$$

This set of parameters are similar to that obtained in eq. (14). But very different than that obtained from using  $B \rightarrow K\pi$  data only.

## VII. CONCLUSIONS

SU(3) flavor symmetry can simplify the analysis for  $B \rightarrow K\pi$  and  $B \rightarrow \pi\pi$ . To the leading order there are only five independent hadronic parameters for these decays in the Standard Model. The leading amplitudes  $T$ ,  $C$  and  $P$  can provide a reasonable description of  $B \rightarrow K\pi$  decays, but with an inexplicably large value of the ratio  $|C/T|$ . When combined with  $B \rightarrow \pi\pi$  data, there are more difficulties. One of the problems is that the ratio  $|C/T|$  is still of order one, and much larger than theoretical estimates. And another is that these parameters can not give a better than a two standard deviation fit to the current data.

As a first step in trying to resolve these difficulties, we studied several possible ways of relaxing approximations made in the simple parametrization, including sub-leading order terms, annihilation, exchange and charming penguin amplitudes, and also SU(3) breaking effects. We find that the inclusion of smaller sub-leading terms can improve the fit to a reasonable range although still results a ratio of order one for  $|C/T|$  which may be due to low energy hadronic physics within the Standard Model. It is too earlier to claim the need of new physics beyond the Standard Model to explain the  $B \rightarrow K\pi$  and  $B \rightarrow \pi\pi$  data.

Finally we would like to make a comment on the source for the large  $\chi^2_{min}$  for the leading parametrization from data quality point of view. We find that the data point  $B(K^-\pi^+)$  makes the largest contribution to  $\chi^2$  in the fitting using both  $K\pi$  and  $\pi\pi$  data. If one removes this data point, the  $\chi^2_{min}$  for the two cases resulting in eqs. (7) and (8) would drop down to 0.05 and 11, respectively. This is because that the experimental branching ratio

for  $K^-\pi^+$  is smaller than expected from penguin dominance in  $K\pi$  decays. The point we would like to emphasize here is that it is necessary to have more precise data to help deciding whether the Standard Model can explain both  $B \rightarrow K\pi$  and  $B \rightarrow \pi\pi$ .

### Acknowledgements

XGH would like to thank H.-Y. Cheng and H.-N. Li for useful discussions. This work was supported in part by the NSC of ROC and the ARC of Australia.

- 
- [1] Babar collaboration, B. Aubert et al., hep-ex/0407057; Belle Collaboration, Y. Chao et al., hep-ph/0408100.
  - [2] M. Giorgi, Plenary talk at ICHEP'04; Y. Sakai, Plenary talk at ICHEP'04.
  - [3] Heavy Flavor Averaging Group [HFAG], <http://www.slac.stanford.edu/xorg/hfag>.
  - [4] D. Zeppenfeld, Z. Phys. **C8**, 77 (1981); L.-L. Chou and H.-Y. Cheng, Phys. Rev. Lett. **56**, 1655 (1986).
  - [5] M. Gronau et al., Phys. Rev. **D50**, 4529 (1994); **D52**, (1995) 6374, C.-W. Chiang, M. Gronau and J. Rosner, Phys. Rev. **D68** (2003) 074012.
  - [6] M. Savage and M. Wise, Phys. Rev. **D39**, 3346 (1998); Erratum 3127 (1998); B. Grinstein and R. Lebed, Phys. Rev. **D53**, 6344 (1996); M. Gronau, D. Pirjol and T.-M. Yan, Phys. Rev. **D60**, 034021 (1999); Erratum **D68**, 119901 (2004).
  - [7] X.-G. He, Eur. Phys. J **C9**, 443 (1999); X.-G. He et al., Phys. Rev. **D64**, 034002 (2001). H.-K. Fu et al., Chin. J Phys. **41**, 601 (2003); H.-K. Fu et al., Phys. Rev. **D69**, 074002 (2004).
  - [8] A. Buras et al., Phys. Rev. Lett. **92**, 101804 (2004); Nucl. Phys. **B697**, 133(2004); C.-W. Chiang et al., Phys. Rev. **D69** 034001 (2004); J. Charles et al., (CKMfitter), arXiv: hep-ph/0406184.
  - [9] Y.-L. Wu and Y.-F. Zhou, arXiv: hep-ph/0409221; C.-K. Chua, H.-Y. Cheng and A. Soni, arXiv: hep-ph/0409317; Y.-Y. Charng and H.-N. Li, hep-ph/0410005.
  - [10] S. Nandi and A. Kundu, arXiv: hep-ph/0407061; S. Khalil and E. Kou, arXiv: hep-ph/0407284; S. Mishima and T. Yoshikawa, arXiv: hep-ph/0408090; X.-G. He, C.-S.



- Li and L.-L. Yang, arXiv: hep-ph/0409338.
- [11] Particle Data Group, Phys. Lett. **B592**, 1 (2004).
  - [12] A. Buras et al., arXiv: hep-ph/0410407.
  - [13] Y.L. Wu and Y.-F Zhou, arXiv: hep-ph/0503077.
  - [14] S. Baek et al., arXiv: hep-ph/0412086; C.S. Kim. S. Oh and C. Yu, arXiv: hep-ph/0505060; S. Khalil, arXiv: hep-ph/0505151; S. Baek and D. London, arXiv: hep-ph/0504142.
  - [15] G. Buchalla, A. Buras and M. Lautenbacher, Rev. Mod. Phys. **bf 68**, 1125 (1996).
  - [16] M. Neubert and J. Rosner, Phys. Lett. **B441**, 403 (1998).
  - [17] M. Imbeault et al., Phys. Rev. Lett. **92**, 081801 (2004).
  - [18] N.G. Deshpande and X.-G. He, Phys. Rev. Lett. **75**, 1703 (1995); M. Dariescu, et al., Phys. Lett. **B557**, 60 (2003).